

Progress in Lattice QCD Relevant for Flavor Physics

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Recent Lattice QCD results relevant for Kaon, Charm and B Physics are summarized. There is general agreement among calculations using a wide range of different lattice actions. This bolsters confidence in the lattice results and in their quoted errors. One notes considerable progress since CKM2008 in reducing lattice errors with some quantities now being calculated at the subpercent to a few percent level accuracy. Much work remains, however, and further improvements can be expected in the coming years.

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Collaboration (yr)	$f_+(0)$	N_f	# lattice spacings	action
RBC/UKQCD (07) [4]	0.964(5)	2 + 1	1	domain wall
RBC/UKQCD (10) [5]	0.960 $^{+5}_{-6}$	2 + 1	1	domain wall
ETMC (09) [6]	0.956(8)	2	3	twisted mass

Table 1: Semileptonic Kaon decay form factor $f_+^{K \rightarrow \pi}(0)$

1 Introduction

A major goal of Lattice QCD is to carry out the theoretical calculations that are necessary and relevant to the Flavor Physics Program in Particle Physics. There has been huge progress in recent years in working towards this goal. The most accurate lattice calculations now typically have errors at the 0.5 – 4% level in Kaon physics ($f_+^{K \rightarrow \pi}(0)$, f_K/f_π , B_K , ...), at the 1 – 10% level in Charm physics (f_{D_s}/f_D , f_D , f_{D_s} , $f_+^{D \rightarrow K}(0)$, $f_+^{D \rightarrow \pi}(0)$, ...) and at the 2 – 10% level in B Physics ($\mathcal{F}(1)$, f_{B_s}/f_B , f_B , f_{B_s} , ξ , $f_+^{B \rightarrow \pi}(q^2)$, ...). It is particularly noteworthy that sub-percent accuracy has been achieved in Kaon physics and that the ability to simulate very light quarks has improved dramatically, with the pion mass approaching the physical value ever more closely. Furthermore, recent years have witnessed significant improvements in our ability to simulate charm quarks accurately on the lattice. Much work remains, however in Lattice Flavor Physics especially in B physics. In this review talk, I would like to present a brief summary of recent results and plans for future improvements. Due to page restrictions, several topics and all details will have to be omitted.

2 Kaon Physics

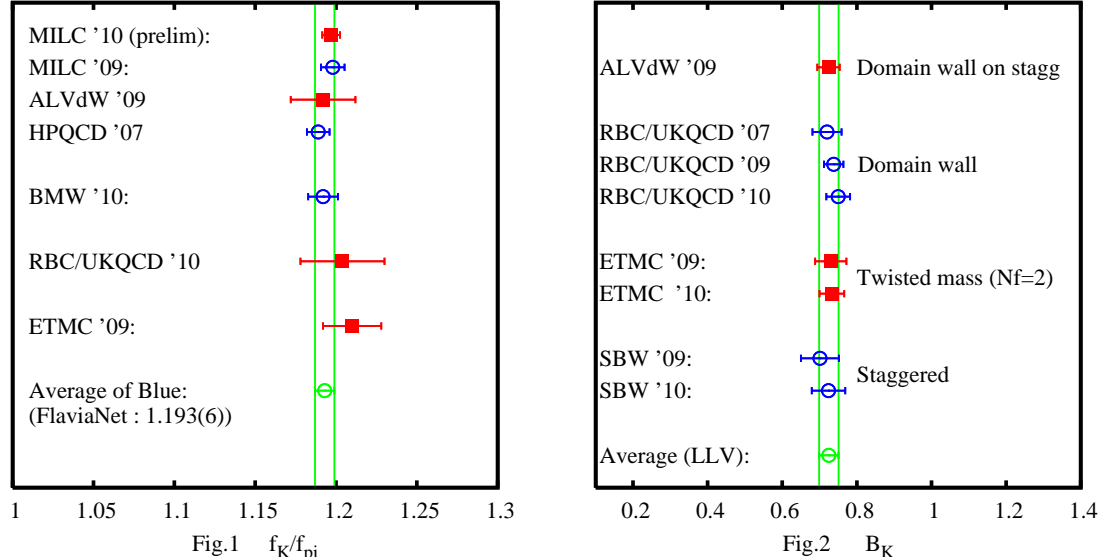
A recent global analysis by FlaviaNet demonstrates the precision with which K_{l3} and K_{l2} decays are testing the Standard Model (SM). For instance, tests of first row unitarity now stand at [1, 2],

$$\Delta_{CKM} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0.0001(6).$$

Lattice QCD input is crucial for determining $|V_{us}|$ [3]. It provides the form factor $f_+^{K \rightarrow \pi}(0)$ for analysis of K_{l3} decays and the ratio of decay constants f_K/f_π for the K_{l2} analysis. Table 1 shows recent lattice results for $f_+^{K \rightarrow \pi}(0)$. The RBC/UKQCD collaboration uses domain wall fermions [4, 5]. Their 2010 update works directly at $q^2 = 0$ and employs a more sophisticated chiral perturbation theory (ChPT) analysis than in their earlier work. In the ETMC calculations based on twisted mass quarks [6], strange sea quark contributions were not included directly but their effects were estimated and included through NLO in ChPT. Fig.1 summarizes lattice results for

the ratio f_K/f_π from many lattice collaborations using a variety of lattice quark actions [7, 8, 9, 10, 11, 12, 13]. An average by the FlaviaNet group, $f_K/f_\pi = 1.193(6)$ [1] is also shown.

There has also been progress on the $K^0 - \bar{K}^0$ mixing parameter B_K . Fig.2 shows a summary and one sees excellent agreement between results from different lattice actions [14, 15, 16, 17, 18]. An average of a subset of the data by Laiho-Lunghi-VandeWater (LLV), $\hat{B}_K = 0.725(26)$ [19], is included in the plot.



The next frontier in Kaon physics is $K \rightarrow \pi\pi$ in the $\Delta I = 3/2$ and $\Delta I = 1/2$ channels [20]. Progress has been made recently by two groups using different approaches. RBC/UKQCD [21] uses a “direct” Lellouch-Luescher finite volume method. Coumbe-Laiho-Lightman-VandeWater [22] first work at an unphysical kinematic point, $2M_\pi = M_K$ in order to evade the Maiani-Testa No-Go Theorem that afflicts Euclidean lattice simulations of this process. They then use ChPT to get back to physical pions. At least for the $\Delta I = 3/2$ case, both methods appear feasible with errors at the 15 - 20% level.

3 Charm Physics

Studies of leptonic decays of the D and in particular of the D_s mesons have attracted considerable attention in the last couple of years. Experiment and theory are compared against each other via the decay constants f_D and f_{D_s} . Table 2 summarizes lattice calculations of the decay constants as of summer 2010. Experimental determinations of decay constants start from the branching fractions for the leptonic decays of charged D_q mesons $\mathcal{B}(D_q \rightarrow l\nu)$, corrected for electromagnetic effects ($q = d$ or s). In the Standard Model these branching fractions depend on the combinations

Collaboration	f_D (MeV)	f_{D_s} (MeV)	f_{D_s}/f_D
Fermi/MILC '05 [23]	201 ± 17	249 ± 16	1.24 ± 0.07
Fermi/MILC '10 [24] (preliminary)	$220 \pm 9 \pm 5$	$261 \pm 8 \pm 5$	1.19 ± 0.01 ± 0.02
HPQCD '07 [10]	207 ± 4	241 ± 3	1.164 ± 0.011
HPQCD '10 [25]		248.0 ± 2.5	
ETMC '09 ($N_f = 2$) [13]	197 ± 9	244 ± 8	1.24 ± 0.03

Table 2: Decay constants f_D and f_{D_s}

$f_{D_q} * |V_{cq}|$. Using estimates of the CKM matrix elements $|V_{cq}|$ based on unitarity one then has, $f_{D_q}^{exp.}$ [26, 27].

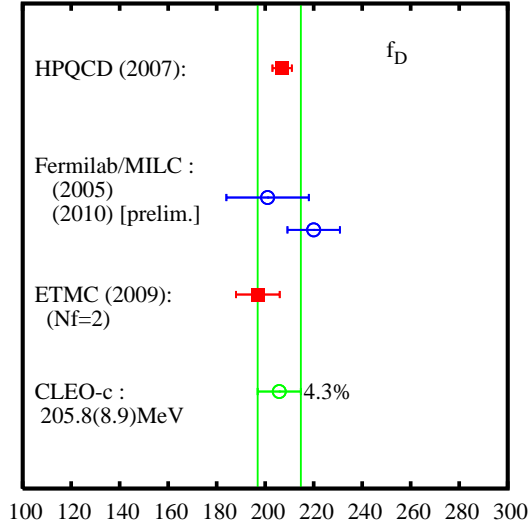


Fig.3: D Meson Decay Constant in MeV

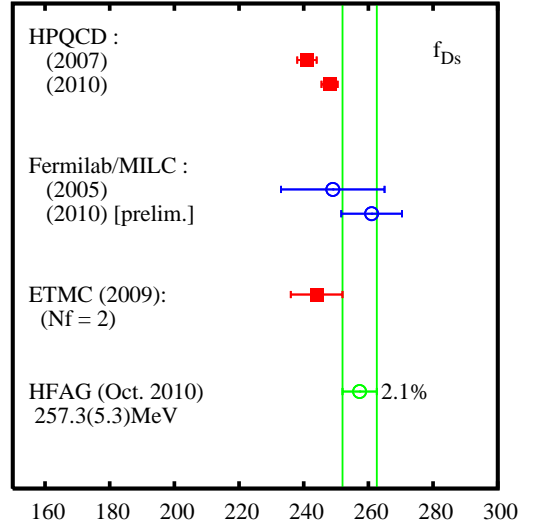


Fig.4: D_s Meson Decay Constant in MeV

Comparisons between experiment and the Lattice QCD results of Table 2 are shown in Figs.3 and 4. For f_D agreement is very good. For f_{D_s} the most accurate 2010 HPQCD result and the HFAG October 2010 experimental average [27] differ by about 1.6σ . It will be interesting to see how things develop in the future when more lattice calculations from other groups and experimental results reach the $\sim 1\%$ level of accuracy.

Significant progress can also be reported on in studies of D meson semileptonic decays on the lattice. Here the goal is to obtain the form factors $f_+^{D \rightarrow K}(q^2)$ and $f_+^{D \rightarrow \pi}(q^2)$ and combine this with experimental measurements of $|V_{cq}| * f_+(0)$ to determine $|V_{cd}|$ and $|V_{cs}|$. A very recent result by the HPQCD collaboration [28] for the $D \rightarrow K$ form factor at $q^2 = 0$ is given below together with $|V_{cs}|$ extracted using experimental

input from CLEO-c [29] and BaBar [30]. Fig.5 plots the HPQCD result together with earlier theory calculations, both from the lattice [31] and using lightcone sum rules methods [32]. The theory error has been reduced by about a factor of four by the 2010 HPQCD calculations. The main reasons for this improvement come from the use of a highly improved lattice action (HISQ) for both the charm and the lighter quarks, the exploitation of hadronic matrix elements that do not require any renormalization and novel approaches to fitting and to chiral/continuum extrapolations of lattice data.

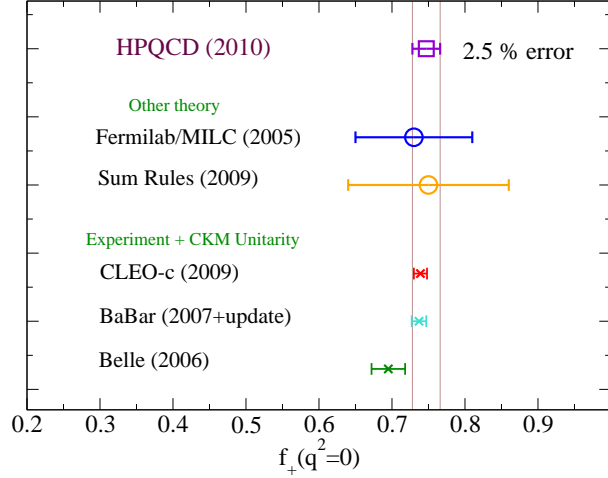


Fig.5

Fig.5 also includes experimental determinations of $f_+^{D \rightarrow K}(0)$ [29, 30, 33]. These were obtained by using the unitarity value for $|V_{cs}|$. One sees that the CLEO-c and BaBar results are already very precise, with errors at the $\sim 1\%$ level a factor of two smaller than for the best lattice calculations. So, inspite of the recent impressive progress on the lattice side, further improvements are called for.

The HPQCD collaboration has used its new $|V_{cs}|$ to test CKM unitarity [28]. Using PDG values $|V_{cd}| = 0.230(11)$ and $|V_{cb}| = 0.0406(13)$ one finds for 2nd row unitarity,

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 0.978(50).$$

Similarly for the 2nd column with $|V_{us}| = 0.2252(9)$ and $|V_{ts}| = 0.0387(21)$ one finds

$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 0.976(50).$$

These unitarity tests are more accurate than the current PDG 2010 results [2] of 1.101(74) and 1.099(74) for the 2nd row or 2nd column respectively.

The Fermilab/MILC collaborations are updating their 2005 D semileptonic form factor calculations with improved statistics and chiral/continuum extrapolations using

HPQCD 2010 [28] :

$$f_+^{D \rightarrow K}(0) = 0.747(19)$$

\Rightarrow

$$|V_{cs}| = 0.961(11)(24)$$

1st error from exper.

2nd error from theory.

many more lattice spacings [34]. Preliminary results for the scaled $f_+(q^2)/f_+(0.15)$ and a comparison with CLEO-c data are given in Fig.6. Fig.7 shows preliminary results by the ETMC collaboration [35]. In both cases the lattice calculations reproduce the shape of the form factor as a function of q^2 in good agreement with experiment. Fermilab/MILC and ETMC are working towards final results for $|V_{cs}|$ and $|V_{cd}|$. HPQCD is also repeating its $D \rightarrow K$ calculations for the $D \rightarrow \pi$ case.

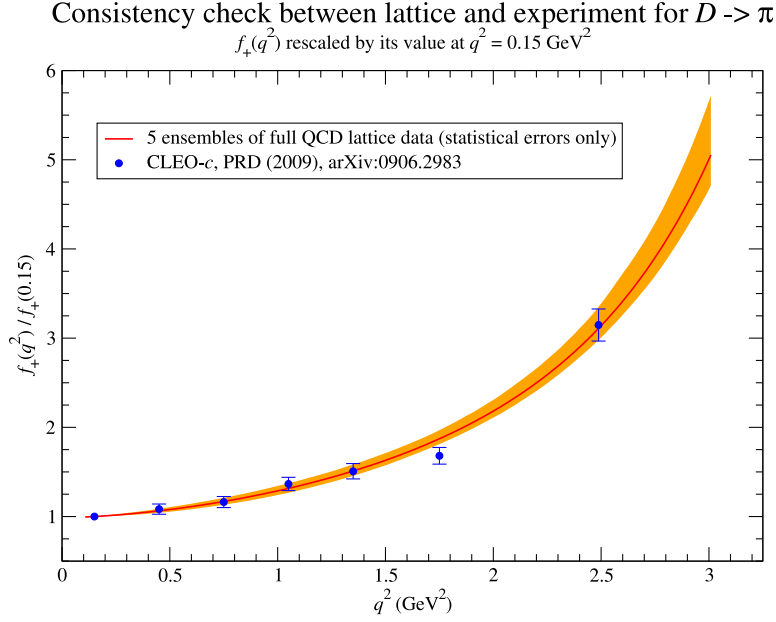


Fig.6: Fermilab/MILC [34]

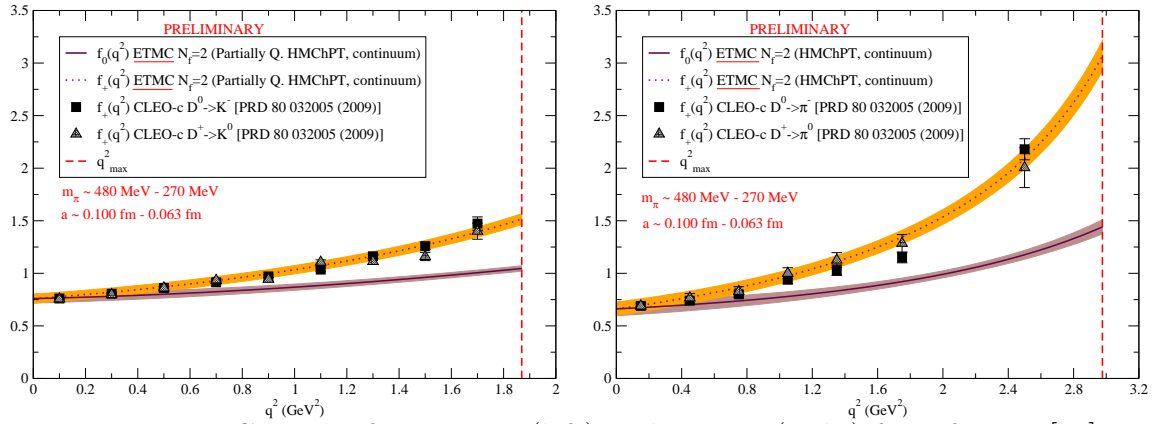


Fig.7: ETMC results for $D \rightarrow K$ (left) and $D \rightarrow \pi$ (right) form factors [35]

4 B Physics

Since CKM2008 there have been updates of the $B \rightarrow D^*$ form factor at zero recoil $\mathcal{F}(1)$, new results for $B - \bar{B}$ mixing and several calculations/updates of the decay constants f_B and f_{B_s} [36]. Fermilab/MILC's results for $\mathcal{F}(1)$ are given below [37, 38]. Fig.8 shows their most recent chiral/continuum extrapolations from 4 lattice spacings.

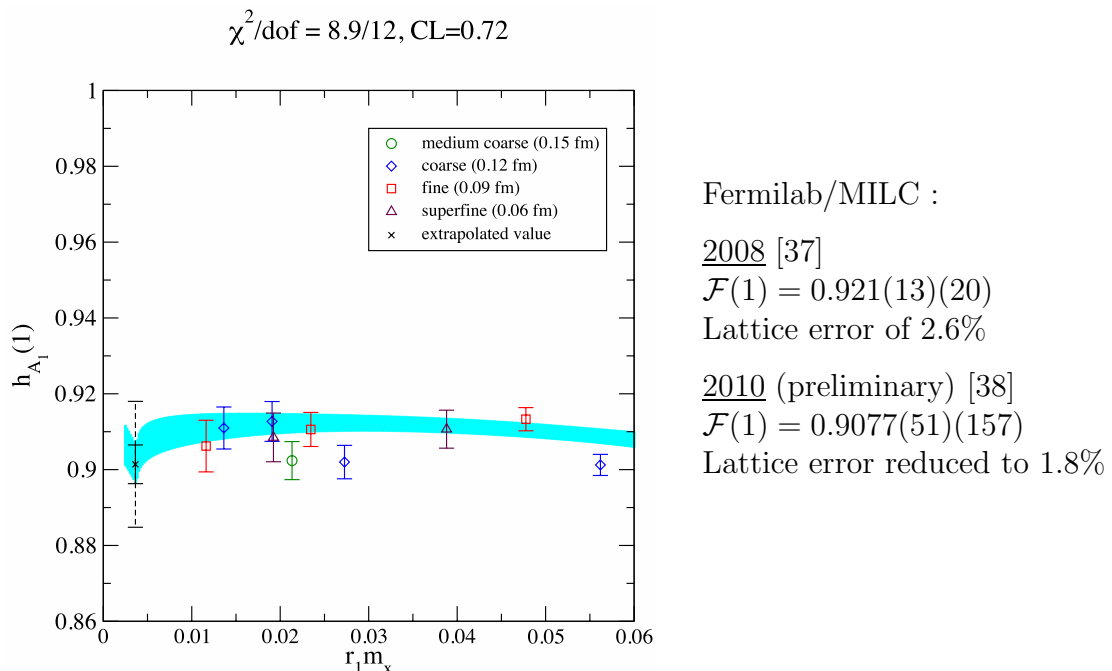


Fig. 8 : Chiral/continuum extrapolation of the $B \rightarrow D^*$ form factor $\mathcal{F}(1)$.

The fact that the central value of $\mathcal{F}(1)$ has come down between 2008 and 2010 means that $|V_{cb}|_{excl.}$ has gone up. At this meeting Mackenzie [39] quotes $|V_{cb}|_{excl.} = 39.7(7)(7) \times 10^{-3}$ using Fermilab/MILC's new $\mathcal{F}(1)$ and an HFAG 09 experimental average for $|V_{cb}| * \mathcal{F}(1)$. This updated $|V_{cb}|_{excl.}$ is to be compared with the PDG 2010 $|V_{cb}|_{incl} = 41.5(7) \times 10^{-3}$. The exclusive and inclusive determinations differ now by just $\sim 1.5\sigma$. As has been pointed out by many authors, reducing uncertainty in $|V_{cb}|$ is of crucial importance in testing the SM via consistency checks on the Unitarity Triangle (UT). Tensions have developed, for instance, between $\sin(2\beta)$, ϵ_K and $\Delta M_s/\Delta M_d$. Analysis of the UT is very sensitive to $|V_{cb}|$. One has $\epsilon_K \propto |V_{cb}|^4$ and hence the uncertainty in ϵ_K is dominated by that of $|V_{cb}|$ (and not by errors in B_K). In order for $|V_{cb}|^4$ to have errors comparable to or better than that of B_K (currently $\sim 4\%$) one would need to know $|V_{cb}|$ with an accuracy of $\leq 1\%$. The lattice community still has more work to do.

Another important area in B Physics where lattice input is crucial is $B - \bar{B}$ mixing.

This is a vast subject and Lattice QCD has so far addressed only parts of it. In order to cover both SM and BSM physics even at lowest order in $1/M$ one needs hadronic matrix elements of a basis of five $\Delta B = 2$ four-fermion operators usually denoted Q_i , $i = 1, 2, \dots, 5$. Most of the published unquenched work has focused on Q_1 , Q_2 and Q_3 , operators relevant for the SM. From these one can calculate bag parameters such as \hat{B}_{B_q} ($q = s$ or d) and the important ratio $\xi = \frac{f_{B_s} \sqrt{B_{B_s}}}{f_{B_d} \sqrt{B_{B_d}}}$. ξ can be combined with the experimentally measured mass differences ΔM_s and ΔM_d in $B_s - \bar{B}_s$ and $B_d - \bar{B}_d$ mixing to determine the ratio of CKM matrix elements $\frac{|V_{td}|}{|V_{ts}|}$. Fig. 9 shows some recent results for ξ [40, 41, 42]. The HPQCD result is used to get $\frac{|V_{td}|}{|V_{ts}|}$. Their value for the bag parameter \hat{B}_{B_s} also leads to the most precise SM prediction to date for the branching fraction $Br(B_s \rightarrow \mu^+ \mu^-)$.

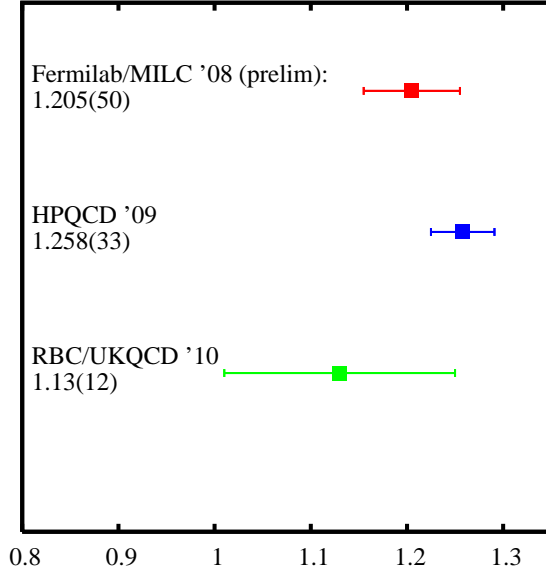


Fig.9: The Ratio xi

$$\text{HPQCD } \xi [41] \implies$$

$$\frac{|V_{td}|}{|V_{ts}|} = 0.214(1)(5)$$

$$\hat{B}_{B_s} = 1.33(6) \implies$$

$$Br(B_s \rightarrow \mu^+ \mu^-) = 3.2(2) \times 10^{-9}$$

Several lattice groups are currently working on improving calculations of four-fermion operator matrix elements. There should be many results coming out soon including matrix elements of operators relevant for BSM physics.

Reference [41] also has updates of B meson decay constants f_B and f_{B_s} . These are shown in Figs.10 and 11 together with preliminary results from the Fermilab/MILC [24] and the ETM [43] collaborations. All groups are working hard on reducing errors. Accurate calculations of f_B are important because of the current tension between lattice and experimental determinations (the latter depends critically on the value used for $|V_{ub}|$).

During the past couple of years work has also been initiated on Rare B Decays. Some preliminary results from the Cambridge group for $B \rightarrow K$ form factors f_+ , f_0 and f_T and for the $B \rightarrow K^*$ form factors T_1 and T_2 were presented at this meeting [44].

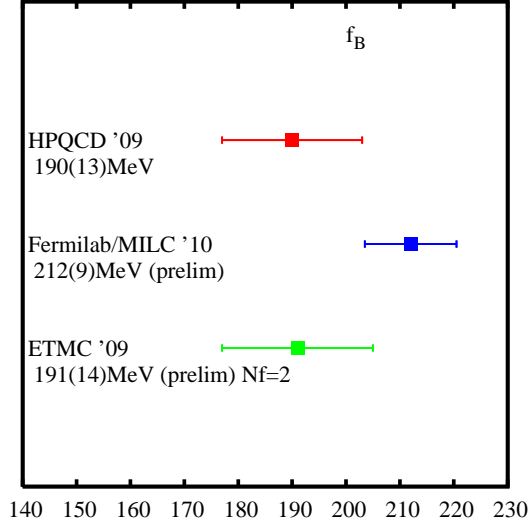


Fig.10: B Meson Decay Constant in MeV

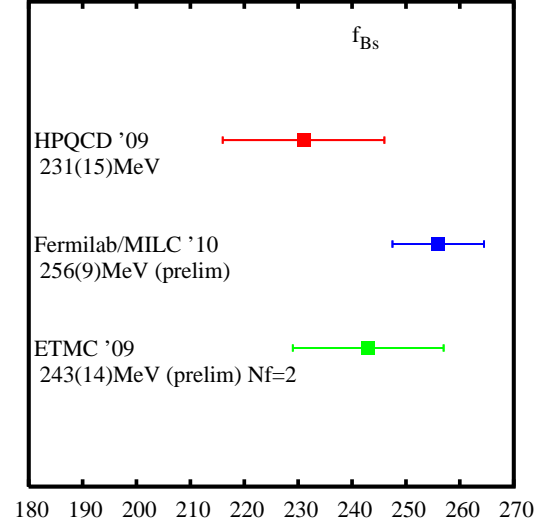


Fig.11: B_s Meson Decay Constant in MeV

5 Summary

It is hopefully clear by now that Lattice QCD is working hard to do its share in Precision Flavor Physics. More and more results with a few % or better errors are becoming available making accurate tests of the Standard Model possible. We look forward to exciting times ahead as we continue to work together with our experimental and theory colleagues.

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